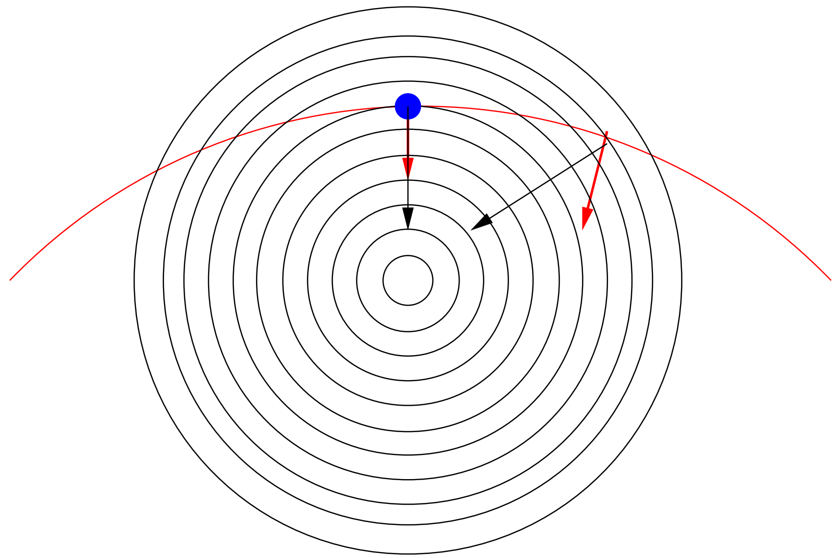
The method of Lagrange multipliers for constrained optimization

In a constrained optimization problem we minimize a function $f(x_1, x_2, ...)$ under the constraint that the minimizing point $x_1^0, x_2^0, ..$ satisfies the constraind $g(x_1^0, x_2^0, ...) = 0$ In the figure below the contour lines of the function f are shown in black and the constrained is shown in red. We assume that the minimum of g is the center of the smallest circle. The constraint curve can also be considered as a contour line where $g = 0$. The gradients of the functions f and g are also shown in black and red. The lowest possible point along the red constrained is obviously the blue point. At this point the red and the black line are tangential and the gradients vectors are collinear, i.e one gradient vector is a multiple of the other one:

$$\nabla f = \lambda \nabla g$$

This equation is the basic equation for a constrained optimization. The quantity λ is called the Lagrange multiplier.



This method of Lagrange multipliers can of course also be used for a constrained maximiation problem, since maximizing a function is identical to minimizing the negative function.